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Title: Early warning signs in social-ecological networks

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17 **Abstract**

18 A number of social-ecological systems exhibit complex behaviour associated with nonlinearities,
19 bifurcations, and interaction with stochastic drivers. These systems are often prone to abrupt
20 and unexpected instabilities and state shifts that emerge as a discontinuous response to gradual
21 changes in environmental drivers. Predicting such behaviours is crucial to the prevention of or
22 preparation for unwanted regime shifts. Recent research in ecology has investigated early
23 warning signs that anticipate the divergence of univariate ecosystem dynamics from a stable
24 attractor. To date, leading indicators of instability in systems with multiple interacting
25 components have remained poorly investigated. This is a major limitation in the understanding
26 of the dynamics of complex social-ecological networks. Here, we develop a theoretical framework
27 to demonstrate that rising variance – measured, for example, by the maximum element of the
28 covariance matrix of the network – is an effective leading indicator of network instability. We
29 show that its reliability and robustness depend more on the sign of the interactions within the
30 network than the network structure or noise intensity. Mutualistic, scale free and small world
31 networks are less stable than their antagonistic or random counterparts but their instability is
32 more reliably predicted by this leading indicator. These results provide new advances in
33 multidimensional early warning analysis and offer a framework to evaluate the resilience of
34 social-ecological networks.

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38 **Introduction**

39 Social-ecological systems are often difficult to investigate and manage because of their inherent
 40 complexity (1). Small variations in external drivers can lead to abrupt changes associated with
 41 instabilities and bifurcations in the underlying dynamics (2-4). These transitions can occur in a
 42 variety of ecological and social systems, and are often unexpected and difficult to revert (4).
 43 Anticipating critical transitions and divergence from the present state of the system is
 44 particularly crucial to the prevention or mitigation of the effects of unwanted and irreversible
 45 changes (5-10). Recent research in ecology has focused on leading indicators of regime shift in
 46 ecosystems characterized by one state variable (5,7,11,12). These indicators are typically
 47 associated with the critical slowing down phenomenon: as the system approaches a critical
 48 transition, its response to small perturbations of the stable state becomes slower (11). It has
 49 been shown that in univariate systems (i.e., with only one state variable) critical slowing down
 50 entails an increase in the temporal variance and autocorrelation of the state variable (5). The
 51 case of systems with several mutually interacting components, however, has remained poorly
 52 investigated (13-15), while the connection between network stability and research on indicators
 53 for loss of resilience has been elusive (16).

54 Here we develop a theoretical framework to analyze early warning signs of instability and regime
 55 shift in complex networks. We provide analytical expressions for a set of precursors of instability
 56 in complex systems with additive noise for a variety of network structures.

57 We consider a social-ecological system with N components (nodes) coupled through a set of
 58 links. The state of the system is expressed by the vector \mathbf{x} of length N , whose terms x_i represent
 59 the state of node i . The local stability of a state \mathbf{x}^* is evaluated through a linearization, $\frac{d\mathbf{y}}{dt} = \mathbf{A} \cdot \mathbf{y}$,

60 where $\mathbf{y} = \mathbf{x} - \mathbf{x}^*$ is the displacement of \mathbf{x} from \mathbf{x}^* ; \mathbf{A} is the $N \times N$ matrix expressing the interactions

among nodes in the (linearized) dynamics (see Methods). In population ecology this framework is typically used to express the dynamics of a community of N populations interacting according to the relationships determined by the matrix \mathbf{A} , often known as "*community matrix*"(2,17-19); likewise, in social systems \mathbf{A} describes the network of interactions (e.g., trade, migration, flow of information among people, groups of individuals, or countries (20-23)). The off-diagonal terms of \mathbf{A} determine the pairs of interacting nodes as well as the strength of their interaction. The dynamics are stable if the maximum real part of the eigenvalues of \mathbf{A} , $\text{Max}[\text{Re}(\lambda)]$, is negative.

Classic ecological theories (2,3) have considered the case of networks with randomly connected nodes (with a certain probability, C). The strength (p) of the interactions between them is represented by a zero-mean random variable of variance σ^2 . May (2,3) showed that random networks become unstable as connectivity (i.e., C), size (i.e., N) or the strength variance increase. The stability of networks with prescribed architectures (e.g., predator-prey, competitive or mutualistic interactions) also depends on connectivity, strength variance, system size, as well as on the network structure (17,19).

More generally, the off-diagonal terms of \mathbf{A} may result from a set of "rules" expressed as a function of a few parameters of which connectivity and strength variance are just an example. Changes in the structure and intensity of the interactions correspond to variations in these parameters, which, in turn, can lead to instability by modifying the community matrix and its eigenvalues. How can we evaluate whether ongoing changes in the interactions within a social-ecological network are reducing its resilience? Is there a way to use measurable quantities to determine whether the system is about to become unstable?

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In one-dimensional systems leading indicators are typically associated with behaviors resulting from the eigenvalue tending to zero at the onset of instability. This effect entails a slower return to equilibrium after a "small" perturbation (11,24). Known as "critical slowing down", this phenomenon exists also in systems with multiple interacting components, though it is hard to recognize and therefore it does not constitute an effective leading indicator of instability. In fact, in "real world" applications the equations driving the dynamics are not known and, therefore, the network nodes in which slowing down is expected to occur are not known a priori. Critical slowing down, however, has been related to an increase in variance and autocorrelation in the state variable of one dimensional systems (5,7,25). Here we provide a theoretical framework to investigate early-warnings in the variance, autocorrelation, and power spectrum of multi-dimensional systems with interactions described by a given network structure.

Methods

We consider a network with N interacting nodes. The state of the system, $\mathbf{x}=\{x_1, x_2, \dots, x_N\}$, is governed by dynamics: $d\mathbf{x} = \mathbf{f}(\mathbf{x}, p, C)dt + \nu \mathbf{I} dW$, where $\mathbf{f}=\{f_1, f_2, \dots, f_N\}$ is a N -dimensional vector function expressing the deterministic component of the dynamics of \mathbf{x} , as a function of a set of parameters, p and C ; \mathbf{I} is the identity matrix, and νdW is an additive stochastic driver represented by a white Gaussian noise of mean zero and intensity νdt . If we consider a small perturbation \mathbf{y} forcing the system away from its equilibrium point \mathbf{x}^* (i.e., $\mathbf{y}=\mathbf{x}-\mathbf{x}^*$), inserting $\mathbf{x}=\mathbf{x}^*+\mathbf{y}$ in the above equation and linearizing $\mathbf{f}(\mathbf{x}^*+\mathbf{y}, p, C)$ around \mathbf{x}^* we obtain

$$d\mathbf{y} = \mathbf{A}(p)\mathbf{y}dt + \mathbf{I}\nu dW, \quad (1)$$

where $A_{ij} = [\partial f_i / \partial x_j]_{\mathbf{x}=\mathbf{x}^*}$. Eq. (1) is a multivariate Ornstein–Uhlenbeck process (26).

107 The stable states, \mathbf{x}^* , of Eq. (1) are the same as those of their deterministic counterparts,
 108 $\frac{dx}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{p}, \mathbf{C})$ (27). These states are stable if the maximum real part of the eigenvalues of \mathbf{A} is
 109 negative. To identify early warning signs of network instability, we relate the steady state
 110 covariance matrix $\mathbf{S}_y = \langle \mathbf{y}_s, \mathbf{y}_s^T \rangle$ to the eigenvalues, λ , of \mathbf{A} , where \mathbf{y}_s is calculated from the steady
 111 state solution of Eq. (1). We first look for leading indicators of instability in the behavior of the
 112 covariance matrix, \mathbf{S}_y , of \mathbf{y} as the system approaches instability. The (i,j) element of \mathbf{S}_y is:
 113 $S_y(i,j) = \langle y_i y_j \rangle - \langle y_i \rangle \langle y_j \rangle$, where $\langle \rangle$ represents the average. The covariance matrix of the
 114 stationary dynamics of the system can be obtained (26) as the solution of Eq. (2):

$$115$$

$$116 \quad \mathbf{A}(\mathbf{p}, \mathbf{C}) \mathbf{S}_y + \mathbf{S}_y \mathbf{A}^T(\mathbf{p}, \mathbf{C}) = -\nu \mathbf{I}. \quad (2)$$

117

118 \mathbf{S}_y is a function of the linearization matrix, $\mathbf{A}(\mathbf{p}, \mathbf{C})$, which, in turn, depends on the control
 119 parameters (\mathbf{p} or \mathbf{C}). At the onset of instability (i.e., as $\text{Max}[\text{Re}(\lambda)] \rightarrow 0$) the maximum element of
 120 the covariance matrix, \mathbf{S}_y , of \mathbf{y} increases. More details on the time-lag correlation and power
 121 spectrum can be found in the Supplementary Materials. While the linearization matrix, \mathbf{A} , here
 122 accounts the interconnections existing among nodes within the network (i.e. the pairs of nodes
 123 that are connected by a link (3, 17)), the covariance matrix, \mathbf{S}_y , expresses the variance of the
 124 fluctuations of the state variable at each node (diagonal terms) and the interrelationship
 125 (positive or negative) of the fluctuations between pairs of nodes (off-diagonal terms). To better
 126 understand the structure of \mathbf{S}_y , we look at the case of a network with only two nodes. In this case
 127 the above equation for the covariance matrix can be solved analytically, and the covariance
 128 matrix reads (26)

$$S_y(p) = \frac{-\nu \text{Det}(\mathbf{A})\mathbf{I} + (\text{Tr}(\mathbf{A})\mathbf{I} - \mathbf{A})\nu\mathbf{I}(\text{Tr}(\mathbf{A})\mathbf{I} - \mathbf{A})^T}{2\text{Det}(\mathbf{A})\text{Tr}(\mathbf{A})},$$

where Det is the determinant and Tr the trace of the matrix, which can be expressed as a function of the eigenvalues, $\lambda_{1,2}$.

Thus, the covariance matrix diverges for $\lambda_1, \lambda_2 \rightarrow 0$. The time correlation matrix, $\rho_y(\Delta)$, can also be computed analytically and also diverges for $\lambda_1, \lambda_2 \rightarrow 0$, independently of Δ . The general analytical expressions of $\rho_y(\Delta)$ and of the power spectrum of \mathbf{y} are also reported in the Supplementary Materials.

We generate networks of size N , with a variety of architectures for \mathbf{A} (see Supplementary Materials), and reach instability either by keeping constant the connectivity, C , while changing the strength of the interactions, p , or by varying C for a fixed p (2,17,19). We then use the analytical relationship between the steady state covariance matrix, \mathbf{S}_y , of \mathbf{y} and the eigenvalues of the matrix \mathbf{A} (Eq. (2)). Similarly, we express the time-lag correlation, ρ_y , and the power spectrum, \mathbf{P}_y , of \mathbf{y} as a function of \mathbf{A} and its eigenvalues.

Results and discussion

We find that the elements of both \mathbf{S}_y and ρ_y increase as the system approaches instability (i.e., $\text{Max}[\text{Re}(\lambda)] \rightarrow 0$). Therefore, we investigate potential indicators for early warning in the behavior of suitable components of \mathbf{S}_y , ρ_y and \mathbf{P}_y for $\text{Max}[\text{Re}(\lambda)] \rightarrow 0$. To that end we first consider the components of \mathbf{S}_y corresponding to the most connected, the most central (28) and the least connected nodes of the network. We also consider indicators based on the properties of the entire network, such as the maximum and the difference between the maximum and minimum of the matrix \mathbf{S}_y .

151 Most of the indicators based on the covariance matrix, \mathbf{S}_y , have a non-trivial dependence on
152 $Max[Re(\lambda)]$ (see Figures 1, S1-S6). The maximum element of \mathbf{S}_y ($Max[\mathbf{S}_y]$) and $Max[\mathbf{S}_y]-Min[\mathbf{S}_y]$
153 provide the most effective indicator of early warning in most networks (Figures 1, S7 and S8). In
154 mutualistic (++) networks $Max[\mathbf{S}_y]$ corresponds to the most connected node (the “hub”),
155 regardless of their topological structure (Supplementary Materials, Figures S9-S10). All these
156 indicators based on \mathbf{S}_y improve their performances when the size, N , of the network increases
157 (compare main panels to insets in Figure 1; see also Supplementary Materials, Figure S11). Thus
158 our ability to detect early warning signs and predict tipping points is enhanced in more diverse
159 systems (16).

160 We also look at the relationship between the maximum element of the time-lag correlation
161 matrix, $\rho_y(\Delta)$ (where Δ is the time lag), and $Max[Re(\lambda)]$ for different values of Δ , p and C (Figures
162 S12-S14). Although significant, these indicators are less efficient with respect to the case with
163 zero time-lag (i.e., indicators based on \mathbf{S}_y). Finally, the power spectrum does not appear to be an
164 effective indicator, as we identified only weak changes in \mathbf{P}_y for increasing values of p and
165 $Max[Re(\lambda)]$ (see Supplementary Materials, Figure S15). Therefore, here we focus on early
166 warning signs provided by the way $Max[\mathbf{S}_y]$ varies as a function of changes in $Max[Re(\lambda)]$.

167 A warning sign is effective if (a) it appears in time to prevent (or prepare for) the occurrence of
168 instability (29-30); (b) it relies on a well-defined and easy to recognize indicator (e.g., a
169 detectable or significant increase in variance (29-30)); and (c) it does not give false positives (or
170 false negatives) (31). We use these criteria to evaluate the effectiveness of $Max[\mathbf{S}_y]$ as a leading
171 indicator of instability with different network structures and levels of noise (32).

172 To investigate the effect of noise, we first consider the “mean-field” case of networks in which the
173 absolute value of the interaction strength between connected nodes is a constant, p ; we gradually

174 increase p or C until $Max[Re(\lambda)]$ becomes positive (17-19). We observe (Figure 2) a consistent
 175 increase in $Max[S_y]$ for all network structures, regardless of whether instability is attained by
 176 increasing interaction strength or connectivity (Figures S1-S6). The network structure, however,
 177 affects the timeliness of $Max[S_y]$ as a leading indicator. In fact, $Max[S_y]$ exhibits a more defined
 178 increase and a better anticipation of the onset of instability in the case of random networks than
 179 with all the other structures. In the case of these “mean field” networks we did not consider the
 180 antagonistic structure because antagonistic networks with constant interaction strength (in
 181 absolute value) are always stable regardless of the parameters p and C (see Supplementary
 182 Materials, Figs. S1, S2).
 183 Likewise, in the case of random interaction strengths $Max[S_y]$ exhibits a well-defined increase
 184 and a better anticipation of the instability in random networks than with the more organized
 185 structures typical of ecological or social systems (Figures 3, S3-S6). The seemingly weaker
 186 increase in $Max[S_y]$ observed in the social-ecological networks is only an apparent effect of the
 187 scale. Indeed, as it will be shown later, suitable detection criteria of early warnings are more
 188 successful in mutualistic networks than in their random counterparts. Noise has the effect of
 189 amplifying the intensity of the warning sign (compare the scales in Figs. 2 and 3), while inducing
 190 weak random fluctuations with no substantial impact on the overall behavior of $Max[S_y]$ at the
 191 onset of instability (see Supplementary Materials). In scale free networks the increase in $Max[S_y]$
 192 (Figure 3) is again only apparently muted. In fact, in these networks detection criteria are quite
 193 successful in recognizing early warning signs (Figure 4); moreover, local indicators (e.g., the
 194 variance of the most central node) can exhibit a more pronounced increase that can be used as
 195 an early warning sign of instability (Figure 1 and Supplementary Materials).

196

197 **Conclusions**

198 We have identified some suitable early warning signs in social-ecological networks in agreement
199 with those identified by Ref. (33), and provided a theoretical framework for their interpretation.
200 Overall, the performances of $Max[S_y]$ as a leading indicator of instability change between random,
201 antagonistic, mutualistic/social networks. This indicator gives an earlier and “sharper” warning
202 sign in random than mutualistic and social networks. The warning sign, however, is harder to
203 detect and is more likely to be missed in random and antagonistic networks than in their
204 mutualistic or social counterparts (Figures 4, S16). Thus, by affecting the probability that early
205 warnings are missed, the sign of the interactions within the network determines the consistency
206 and reliability of this leading indicator. In fact, different realizations of the same network
207 dynamics can yield different results in the behavior of $Max[S_y]$ and thus this indicator might not
208 detect in useful advance the emergence of instability (Figures S17-S18). The probability of true
209 positives is close to 100% (i.e., negligible probability of false negatives) in mutualistic networks,
210 and much smaller in random and antagonistic (predator-prey, cascade or compartment)
211 networks (Figures 4, S16). Thus, while mutualistic networks are less stable than their
212 antagonistic counterparts (17), their instability can be predicted with less uncertainty. An
213 increase in $Max[S_y]$, however, would not provide information on how close the system is to the
214 onset of instability. Rather, it would just indicate that the system is losing resilience and
215 approaching unstable conditions (29). Therefore, in contrast to previous expectations (16), it is
216 not the heterogeneity in the topology of the network that plays a key role in the abruptness of
217 critical transitions and our ability to predict them. Rather, it is the type of interactions between
218 the nodes that determines how networks respond to external perturbations. In fact, there is a
219 trade-off between local and systemic resilience: mutualism (++) is associated with a reduced

220 local stability and resilience of the system (17,19), but does not induce abrupt critical transitions.
221 In contrast, networks with mixtures of interaction types (+-, ++, --) exhibit shorter recovery times
222 after displacement from equilibrium (i.e., a stronger local resilience) (17-18), but in these
223 systems the emergence of systemic instability and critical transitions is more difficult to predict
224 in useful advance.
225 This study combines stability theories from community ecology (2,17) to recent research on
226 indicators of critical transition (7,9,16), and develops a unified framework that offers a new
227 perspective for the evaluation of the resilience and anticipation of instability in social-ecological
228 networks.
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Figures

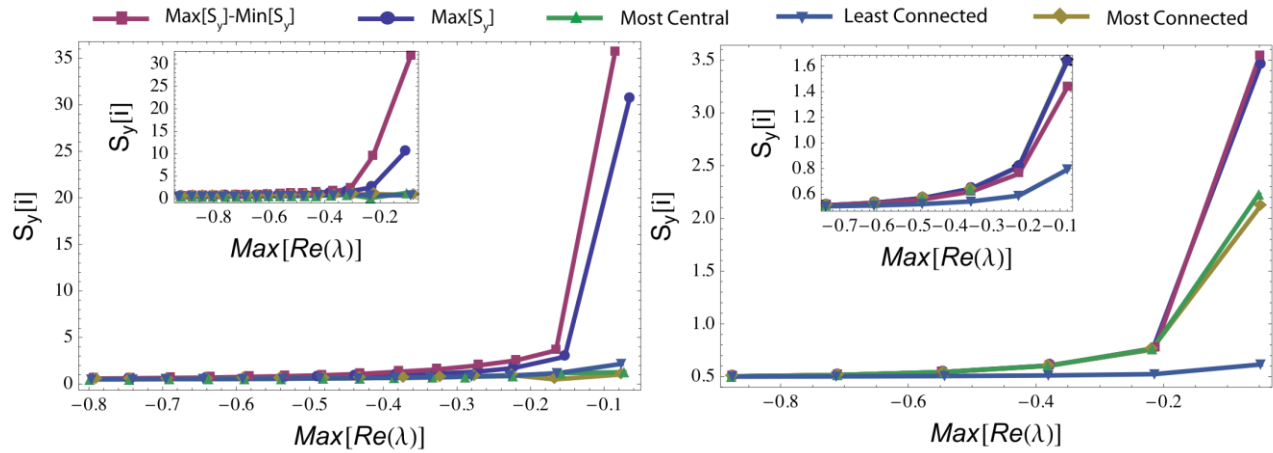


Figure 1. Leading indicators of instability based on different elements of the covariance matrix (\mathbf{S}_y), including the maximum (in absolute value) element, $\text{Max}[\mathbf{S}_y]$, the difference between $\text{Max}[\mathbf{S}_y]$ and $\text{Min}[\mathbf{S}_y]$, the element of \mathbf{S}_y corresponding to the most connected, least connected, or highest eigenvector centrality (24) network node. Random (left) and scale free (right) (30) network generated with $N=50$ and $C=0.1$ (main panels) and $N=0.1$ and $C=0.5$ (insets). Instability (i.e., decrease in $\text{Max}[\text{Re}(\lambda)]$) is attained by increasing the interaction strength p (mean field case). The figures represent average behavior over 100 realizations.

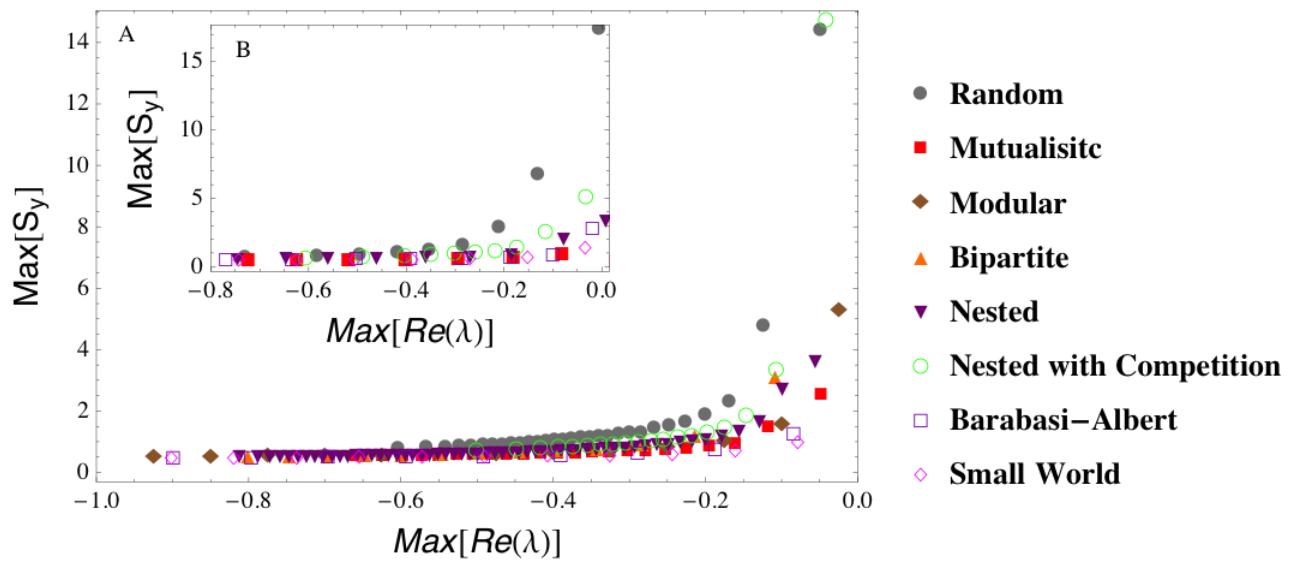


Figure 2. $Max[S_y]$ as a leading indicator of instability in a “mean field” network with constant interaction intensity (in absolute value), p . Instability is attained by increasing p (main panel A, with $N=20$, $C=0.2$) or C (inset B, with $N=20$, and C increasing from 0.1 to 1) with different network structures. The figures represent average behavior over 1000 realizations.

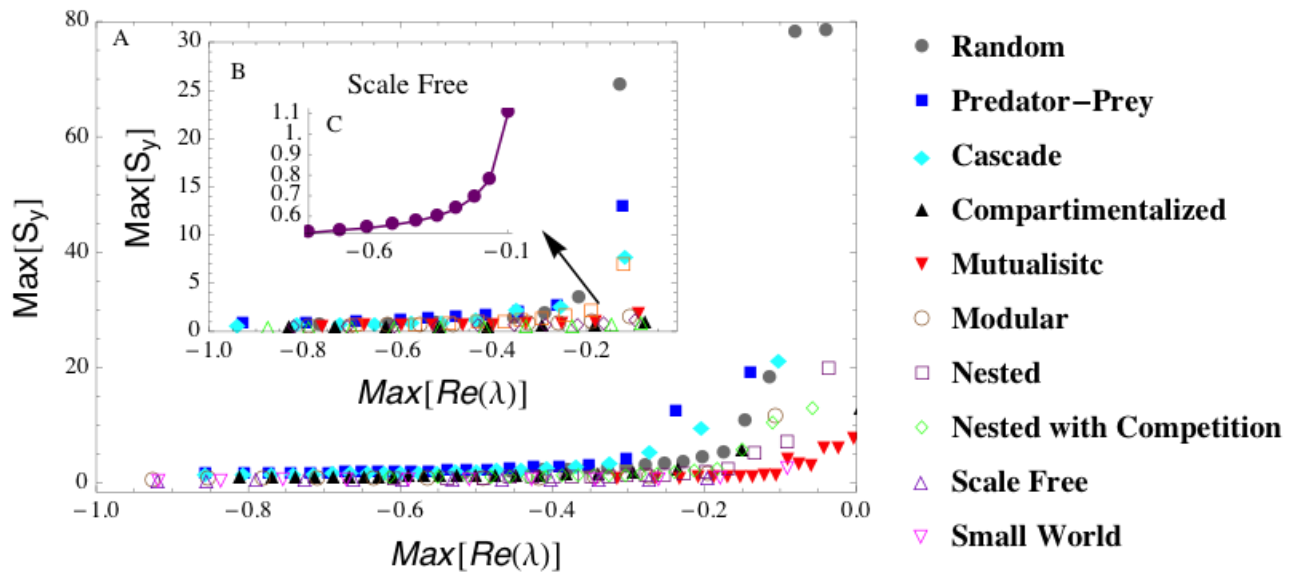
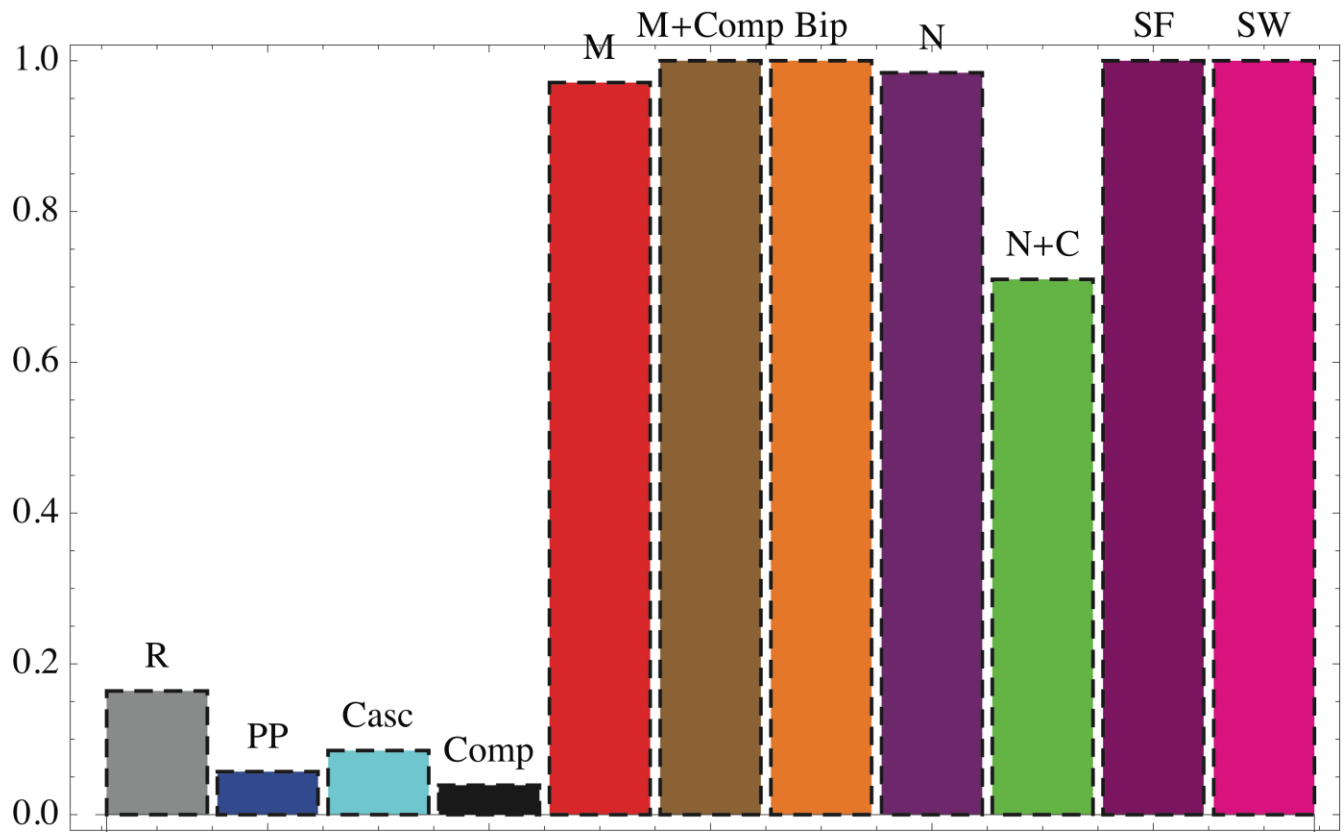


Figure 3. A) Case with random interaction strength (see methods). Main panel: instability is reached by increasing p (with $N=20$; $C=0.2$). First inset (B): p is constant while C increases between 0.1 and 1. C) Same as the first inset (B) but only for the scale-free network (notice the different scale on the vertical axis). The figures represent average behavior over 1000 realizations.



338

339 **Figure 4.** Distribution of the correlation, ρ_K , between $Max[S_y]$ and the parameter p , after 1000
 340 realizations for the full disordered (not mean-field) case. If ρ_K is significant (p -value <0.05) and ρ_K
 341 > 0.5 the increase in $Max[S_y]$ is interpreted as an early warning sign. We calculate these detection
 342 statistics for several realizations of each network structure and determine the probability of
 343 detecting the early warning sign of instability. We consider eleven different network
 344 architectures typical of ecological or social networks, including random (R), predator-prey (PP),
 345 cascade (Casc), compartmentalized (Comp), mutualistic (M), bipartite (Bip), nested (N), nested
 346 with competition (N+C), scale free (SF), and small world (SW). These networks have different
 347 structures for the adjacency matrix and different combination of interaction types, i.e (++)
 348 mutualistic, (+-) antagonistic, (--) competitive or a combination of them (See Supplementary
 349 Materials for more details).

1 **Supplementary online materials**

2 **S1. Structures of Socio-Ecological Interacting systems.**

3 a) *Random matrix* (1) with connectivity C . We pick an element $A_{i,j}$ at random, and with
4 probability C we assign a link between nodes i - j and j - i ($A_{j,i}$). Each of these two links has a
5 weight (or strength), p , that is positive (or negative) with probability 0.5; b) *Antagonistic*
6 *matrix* (2), where connections are again random (with probability C), but if $A_{i,j}$ has sign $+(-)$,
7 then $A_{j,i}$ has sign $- (+)$; c) *Cascade network* (3): links occur with a given probability, but form a
8 hierarchical structure, whereby there is a top “predator” (with sign $+$) that feeds on all the
9 other species (with sign $-$); then there is the second top predator, and so on; species left with
10 the lowest ranking function as producers; d) *Compartmentalized* structure (5), formed by
11 groups of antagonistic species that interact only with species within their own group; e)
12 *Mutualistic network* (6), with a random matrix, but where both $A_{i,j}$ and $A_{j,i}$ have positive signs
13 $(++)$; f) *Modular mutualistic* matrix (5): nodes are divided into communities that positively
14 interact within their groups $(++)$. g) *Bipartite Mutualistic* matrix (6): nodes are divided into
15 two groups, and each node positively interacts only with nodes of the other group $(++)$; h)
16 *Bipartite Nested Mutualistic* (7): a bipartite graph is generated with hierarchical structure
17 where specialist nodes (i.e., with only few mutualistic links) tend to interact with a suitable
18 subset of the mutualistic partners of the generalist nodes $(++)$; i) *Nested Mutualistic with*
19 *Competition* (8): nodes are divided into two groups; each node positively interact with nodes
20 of the opposite group $(++)$, while compete with nodes of the same group $(--)$; j) *Barabasi-*
21 *Albert (BA) networks* (9): binary one-zero networks are generated with power-law degree
22 distribution. This algorithm simulates a preferential attachment process, in which a new
23 vertex with d edges is added at each step. The BA graph displays a scale-free behavior that
24 strongly correlates with the network's robustness to failure. We then assign to each link a

25 positive weight p ; The connectivity of the BA networks is controlled indirectly by the
26 parameter d . In our simulation we set $d=1$ to generate low connectivity networks, $d=2$ for
27 average connectivity and $d=4$ for high connectivity. k) *Watts-Strogatz (WS) network* (10): this
28 is a one-parameter (degree of disorder r) model that interpolates between an ordered finite
29 dimensional lattice and a random graph. Its main property is to display both high clustering
30 coefficient and small world property. The WS model displays this duality for a wide range of
31 the rewiring probabilities r . In our simulation we used $r=0.3$. The connectivity of the WS
32 model depends on the starting $2k$ -regular graph (10). For low connectivity we set $k=2$; for
33 average connectivity $k=4$; for high connectivity $k>5$.

34 The linkage density in each network is described by the parameter C (the connectivity).

35 We generate each of the structures (a)-(k) keeping constant the connectivity C and changing
36 the strength of the interactions (alternatively, we fix p and vary C), until the dynamics become
37 unstable (1,6). Thus, we use the parameter, p , as an indicator of the weights of the interactions
38 between connected nodes (connectivity). We consider three different levels of disorder in the
39 control parameter p : 1) *Mean Field*: where we assign a constant value, p . We do not consider
40 antagonistic mean field networks (i.e., with constant $|p|$ but $(+/-)$ interactions) because they
41 are always stable (see below); 2) *Weak disorder*: the interaction strength is drawn from a
42 normal distribution with mean p and standard deviation $0.1 p$ (i.e., $\mathcal{N}(p, 0.1 p)$); 3) *Strong*
43 *disorder*: interaction strengths are drawn at random from $\pm|\mathcal{N}(0, p)|$ (6). Therefore, in all
44 these cases, by increasing the value of p (C) the resilience decreases until the system become
45 unstable for $p=p_c$, or $C=C_c$ (6).

46

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S2.1 Early Warnings in Multidimensional Mean Field Networks

In the mean field case with (++) interactions, the matrix is symmetric ($\mathbf{A}=\mathbf{A}^T$) and therefore all eigenvalues are real numbers. In this case we note that, if \mathbf{U} is the matrix of the eigenvectors of \mathbf{A} (i.e. $\mathbf{A} \mathbf{u}_i = \lambda_i \mathbf{u}_i$), then $\mathbf{U} \mathbf{A} \mathbf{U}^T = \mathbf{diag}(\lambda)$, where $\mathbf{diag}(\lambda)$ is a diagonal matrix with all the eigenvalues of \mathbf{A} . Therefore in this case we can write Eq. (2) in the main text as:

$$\mathbf{U} \mathbf{A} \mathbf{U}^T \mathbf{U} \mathbf{S}_y \mathbf{U}^T + \mathbf{U} \mathbf{S}_y \mathbf{U}^T \mathbf{U} \mathbf{A} \mathbf{U}^T = -\nu \mathbf{I} = \mathbf{diag}(\lambda) \mathbf{\Sigma} + \mathbf{\Sigma} \mathbf{diag}(\lambda),$$

where $\mathbf{U} \mathbf{U}^T = \mathbf{I}$ and $\mathbf{\Sigma} = \mathbf{U} \mathbf{S}_y \mathbf{U}^T$ is a diagonal matrix whose i -th eigenvalue is $\zeta_i = -\frac{\nu}{2\lambda_i}$.

Therefore if the socio-ecological system reaches an instability (i.e. $\lambda_i=0$) for a given i , then there is at least one element of the matrix $\mathbf{\Sigma}$ and thus of \mathbf{S}_y that diverges.

We also notice that in the mean field case we omit the analysis of early-warning performed on antagonistic (+-) matrices, \mathbf{A}' . In fact, if \mathbf{A}' is anti-symmetric (i.e., $\mathbf{A}' = -\mathbf{A}'^T$, and thus with predator prey, cascade or compartmentalized interactions), then the early warning analysis on those matrices is trivial as their stability does not depend on p or C . In fact, if we multiply the off-diagonal terms of those matrices by the imaginary unit i , i.e. $\mathbf{B} = i(\mathbf{A}' - \mathbf{I} \mathbf{A}')$, then \mathbf{B} is Hermitian and has all real eigenvalues. Therefore $\mathbf{A}' - \mathbf{I} \mathbf{A}'$ has all pure complex eigenvalues independently of p and C , which means that \mathbf{A}' is always stable – given that the self interaction terms are negative constant, i.e. $A'_{i,i} = -d < 0$.

Figures S1 and S2 show a rise in $\text{Max}[\mathbf{S}_y]$ as the system approaches instability in mean field networks.

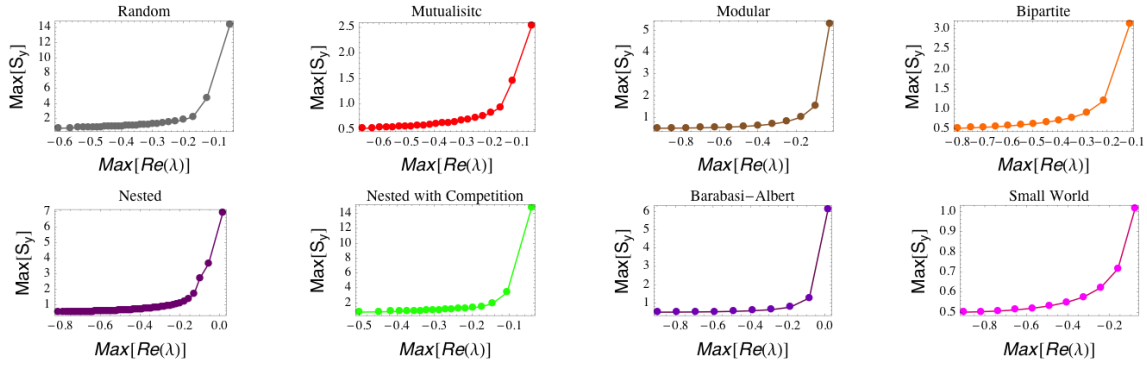


Figure S1. Increase in $Max[S_y]$ as $Max[Re(\lambda)]$ tends to zero for mean field networks of size, $N=20$, $C=0.2$. Increasing values of $Max[Re(\lambda)]$ are obtained by increasing the interaction strength, p . The plotted values are the ensemble averages of 1000 realizations.

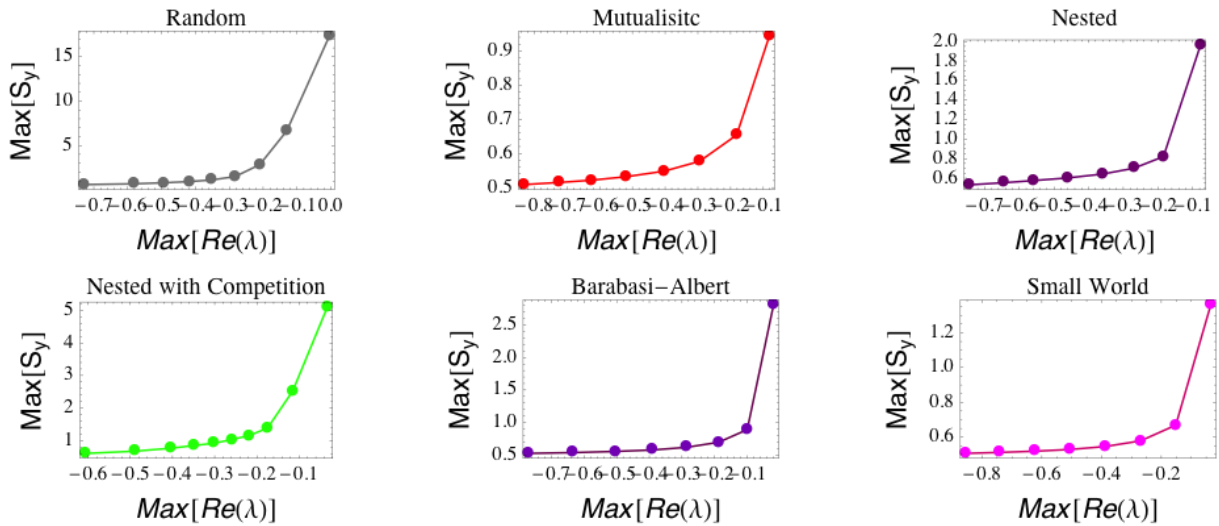
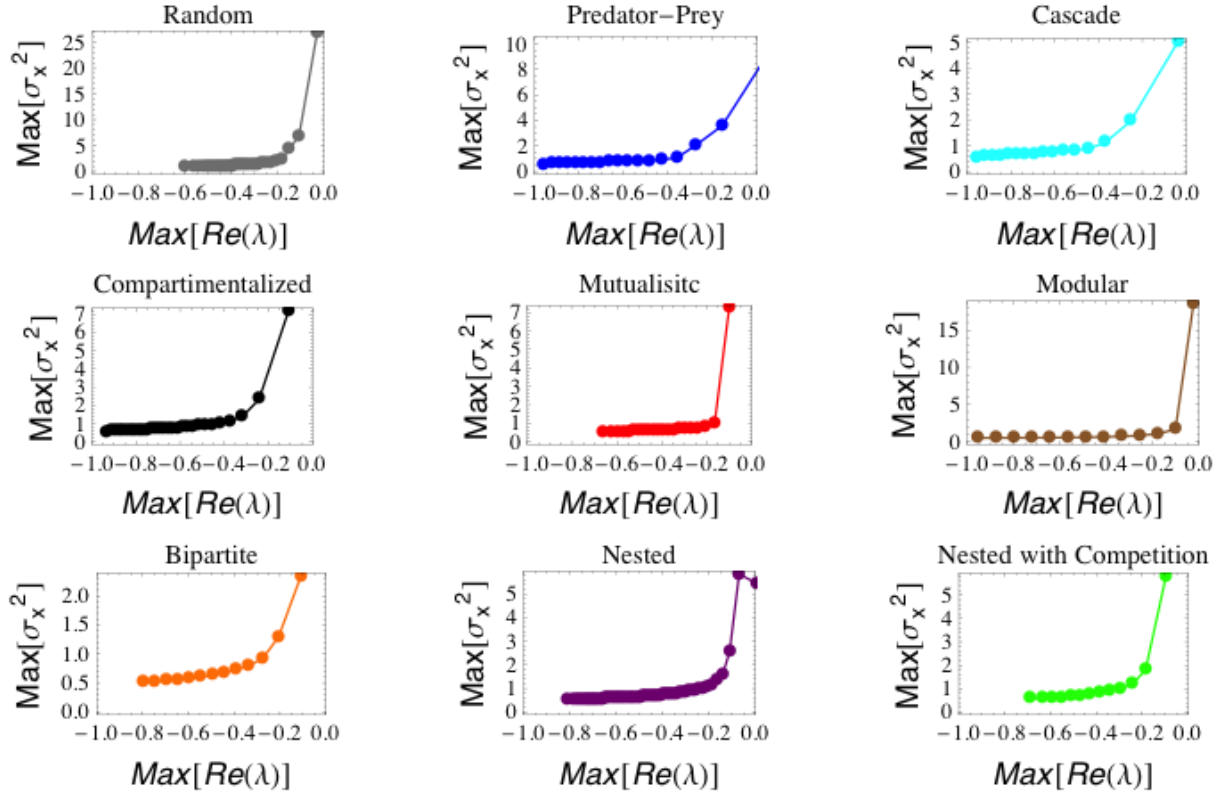


Figure S2. Increase in $Max[S_y]$ as $Max[Re(\lambda)] \rightarrow 0$ for mean field networks of size, $N=20$, $p < p_c$. Increasing values of $Max[Re(\lambda)]$ are obtained by increasing the connectivity, C . The plotted values are the ensemble averages of 1000 realizations.

85 S2.2 Early Warnings in Complex Networks with Weak Disorder



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87

88 **Figure S3.** Increase in $\text{Max}[\sigma_y]$ as $\text{Max}[\text{Re}(\lambda)]$ tends to zero for complex networks with “weak”
 89 disorder (see Section 1) of size, $N=20$ and $C=0.2$. Increasing values of $\text{Max}[\text{Re}(\lambda)]$ are obtained
 90 by increasing the interaction strength, p . The plotted values are the ensemble averages of
 91 1000 realizations.

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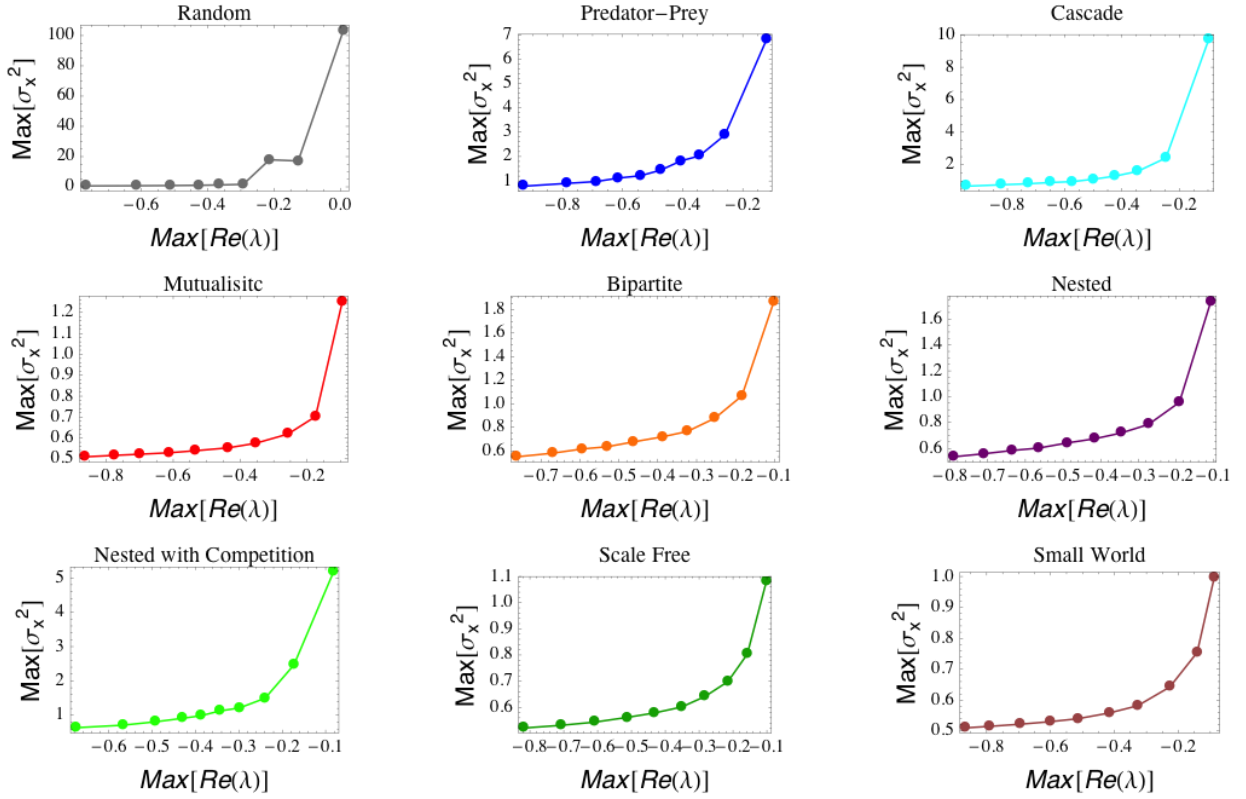
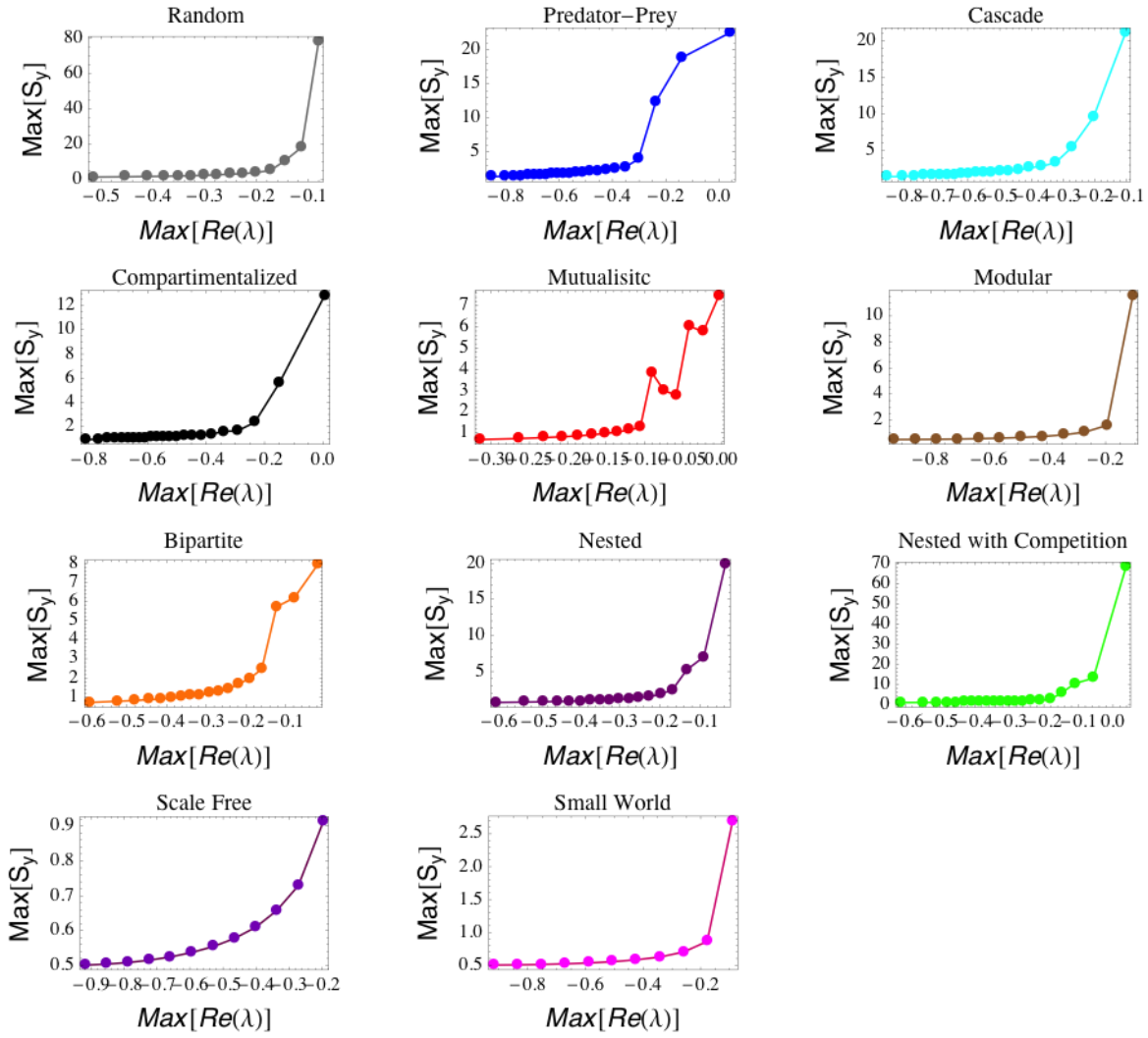


Figure S4. Increase in $\text{Max}[\mathbf{S}_y]$ as $\text{Max}[\text{Re}(\lambda)] \rightarrow 0$ for complex networks with “weak” disorder (see Section 1) of size, $N=20$ and $p < p_c$. Increasing values of $\text{Max}[\text{Re}(\lambda)]$ are obtained by increasing the connectivity, C . The plotted values are the ensemble averages of 1000 realizations.

103 S2.3 Early Warnings in Complex Networks with Strong Disorder



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105

106 **Figure S5.** Increase in $\text{Max}[S_y]$ as $\text{Max}[\text{Re}(\lambda)]$ tends to zero for complex networks with
 107 “strong” disorder (see Section 1) of size, $N=20$ and $C=0.2$. Increasing values of $\text{Max}[\text{Re}(\lambda)]$ are
 108 obtained by increasing the interaction strength, p . The plotted values are the ensemble
 109 averages of 1000 realizations.

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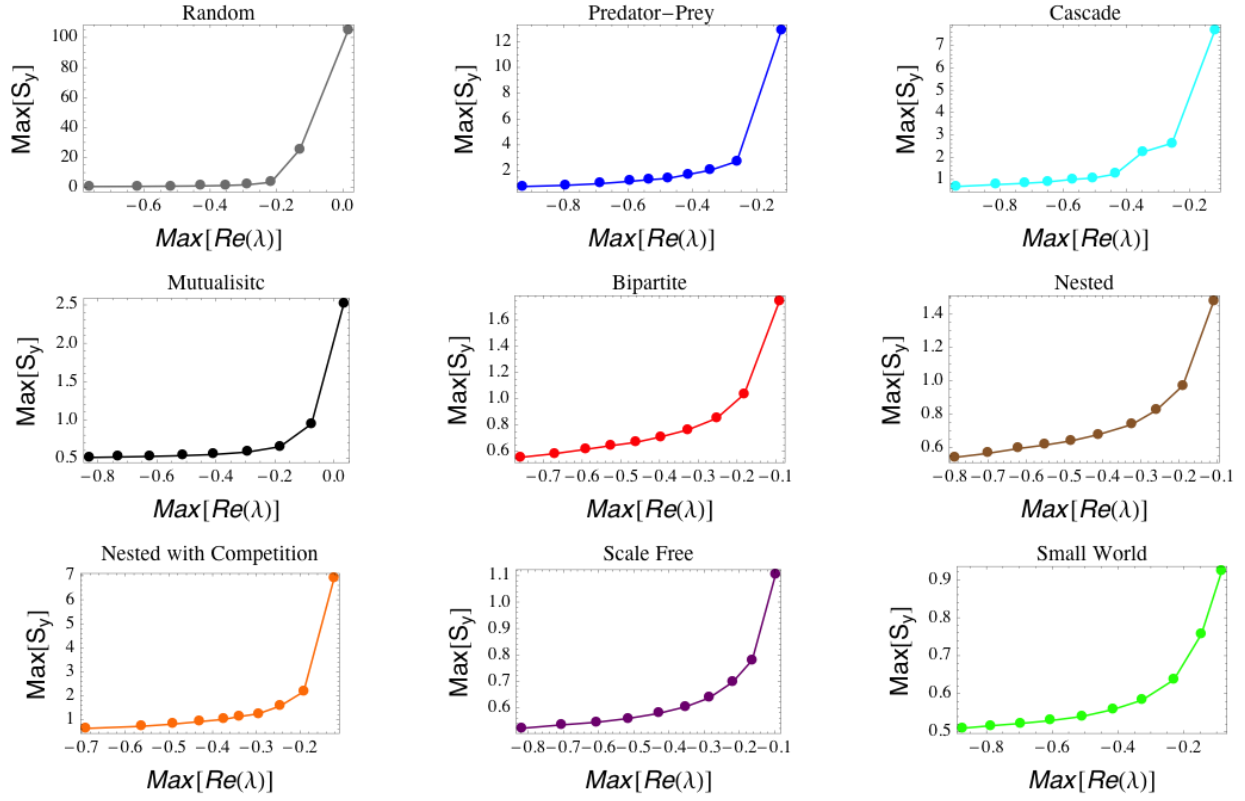


Figure S6. Increase in $Max[S_y]$ as $Max[Re(\lambda)]$ tends to zero for complex networks with “strong” disorder (see Section 1) of size, $N=20$. Increasing values of $Max[Re(\lambda)]$ are obtained by increasing the connectivity, C . The plotted values are the ensemble averages of 1000 realizations.

S3. Indicators and Node Properties

In this section we show more analysis on the effectiveness of different node variances as precursors of instability.

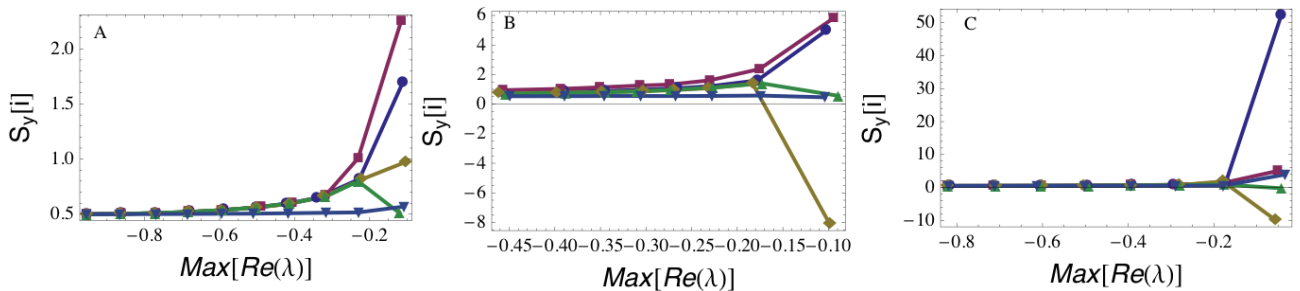


Figure S7. Elements of the covariance matrix \mathbf{S}_y corresponding to nodes with the highest number of connections (green), lowest number of connections (light blue), highest eigenvector centrality (gold), $\max[\mathbf{S}_y]$ (violet) and $\max[\mathbf{S}_y]-\min[\mathbf{S}_y]$ (purple), in the case of: (A) mutualistic, (B) mutualistic nested with competition, (C) small world interactions, for mean field networks (of size $N=20$ and connectivity $C=0.3$). The plotted values are the ensemble averages of 1000 realizations.

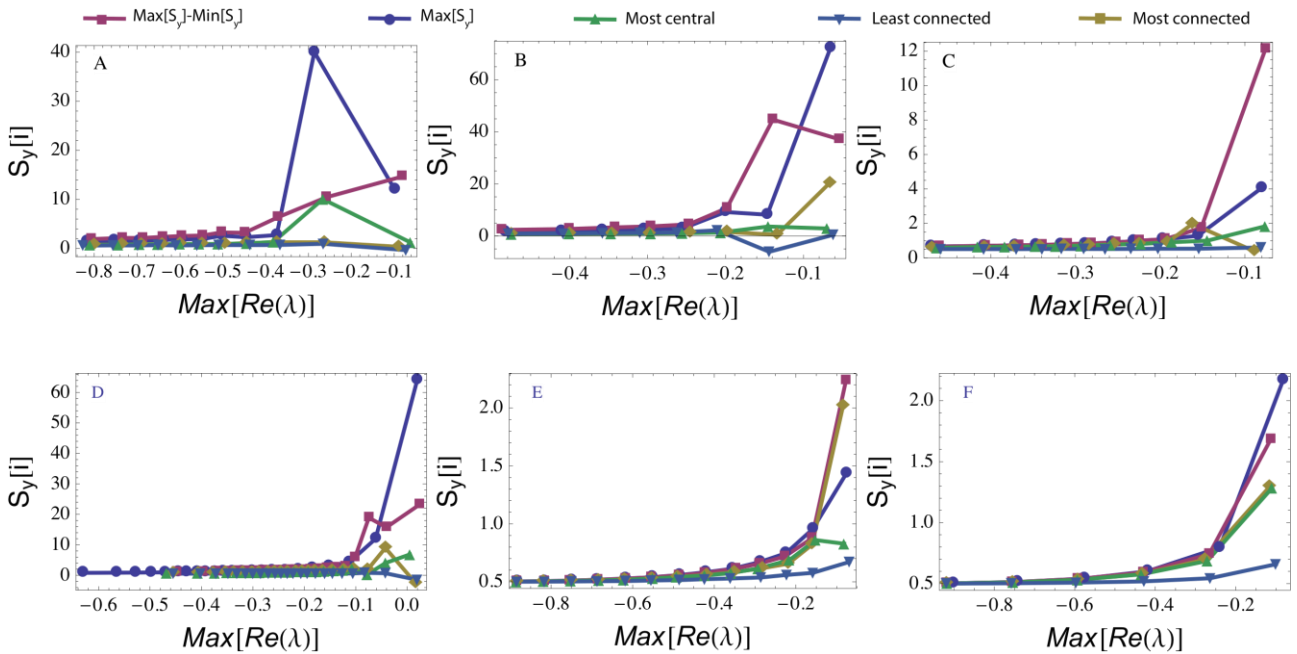


Figure S8. Elements of the covariance matrix \mathbf{S}_y corresponding to nodes with the highest number of connections (green), lowest number of connections (light blue), highest eigenvector centrality (gold), $\max[\mathbf{S}_y]$ (violet) and $\max[\mathbf{S}_y]-\min[\mathbf{S}_y]$ (purple) in the case of (A) random, (B) predator-prey, (C) mutualistic, (D) mutualistic nested with competition, (E) Small world, (F) Barabasi-Albert, networks with strong disorder (of size $N=20$ and connectivity $C=0.3$). The plotted values are the ensemble averages of 1000 realizations.

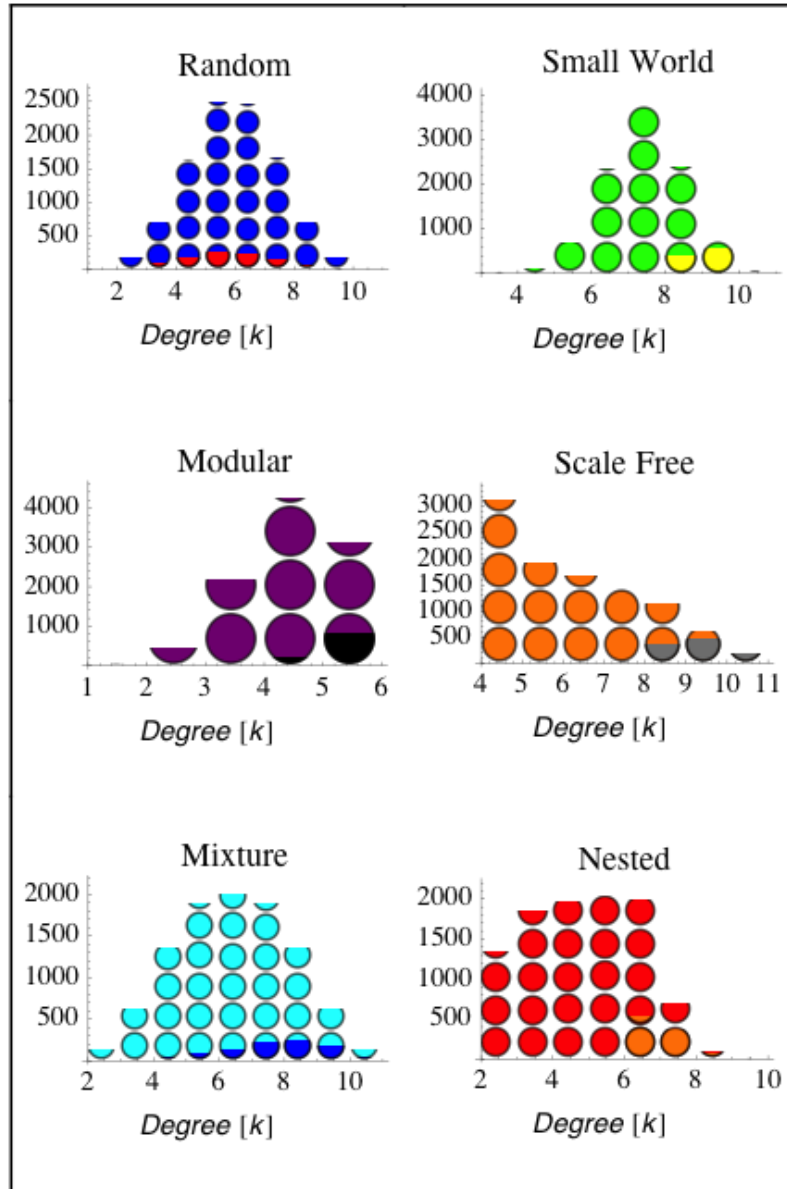


Figure S9. Frequency distribution of the degrees (i.e., number of connections) of the networks' nodes and (with partially filled circles) of the nodes associated with the maximum value of the covariance matrix S_y in mean field networks with a variety of interactions. Based on a set of 100 realizations. Notice how, in mutualistic networks the node corresponding to $\max[S_y]$ is associated with the nodes with the highest degrees (i.e. the generalist species).

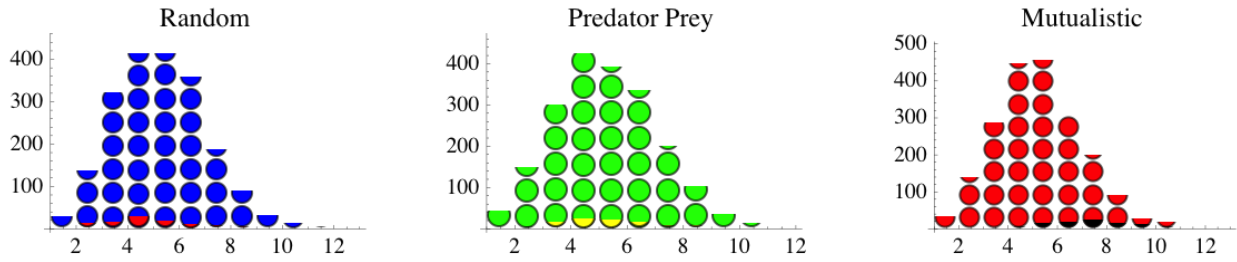


Figure S10. Frequency distribution of the degrees (i.e., number of connections) of the networks' nodes, and (with partially filled circles) frequency distribution of the nodes associated with the maximum value of the covariance matrix \mathbf{S}_y in “strongly” disorganized networks with a variety of interactions. Based on a set of 100 realizations. Notice how, in mutualistic networks the node corresponding to $\max[\mathbf{S}_y]$ is associated with the nodes with the highest degrees (i.e. the generalist species).

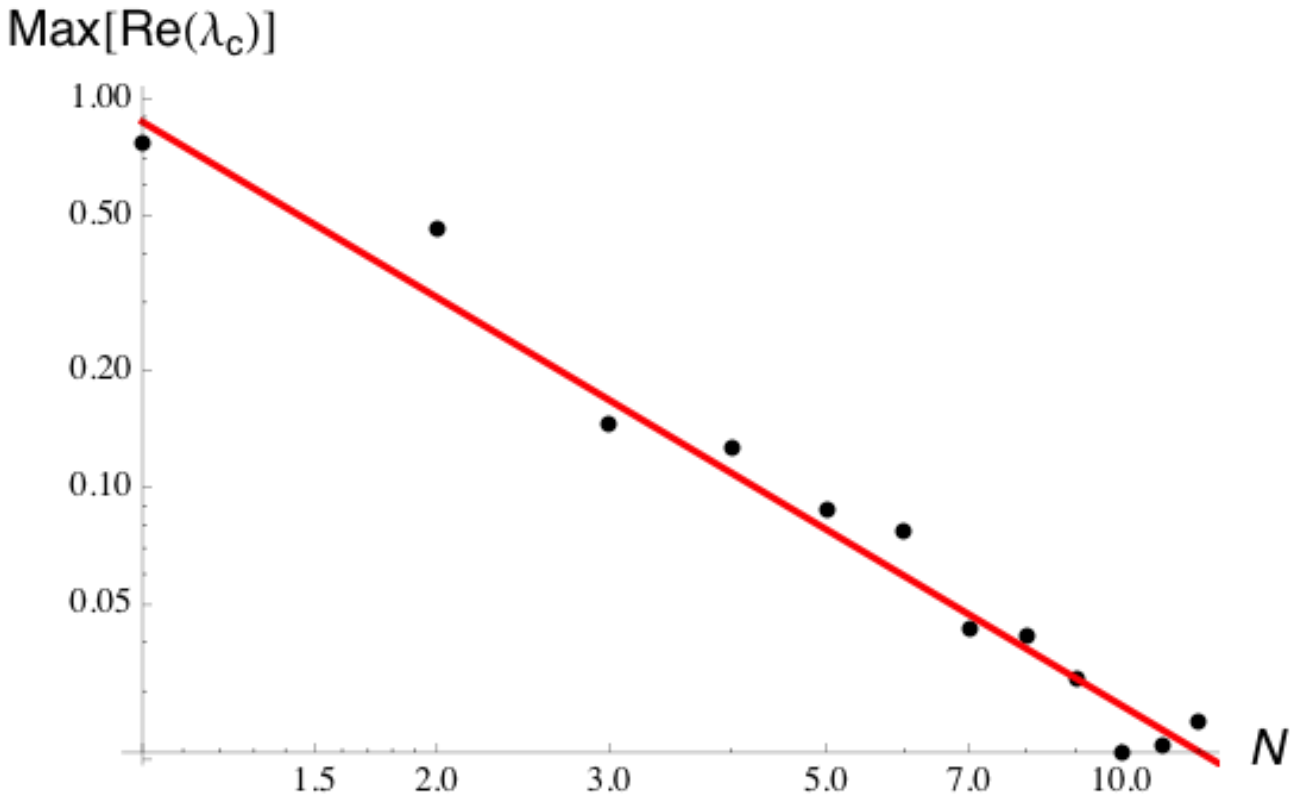


Figure S11. Effect of the network size on the magnitude of the early warning sign. Maximum real part (in absolute value) of the network's eigenvalues as a function of the network size, N for a random network with $C=0.25$ and $p=p_c=1/\sqrt{NC}$. As N increases the max of $\text{Re}(\lambda)$ tends to zero as $\text{Max}[\text{Re}(\lambda)] \sim N^{-1.5}$ and the resilience of the system decreases, while the "height" of the early warning increases. Therefore, as N increases, the early warning sign becomes sharper (see also Figure 1 in the main text).

S4. Early Warning for Time Correlation Matrix and Power Spectrum

We calculate the time lag correlation matrix, $\rho_y(\Delta)$ (where Δ is the time lag), and the power spectrum, $\mathbf{P}_y(\omega)$ (where ω is the frequency) for the steady state dynamics:

$$\rho_y(\Delta) = \langle \mathbf{y}_s(t), \mathbf{y}_s^T(t + \Delta) \rangle = \sigma_y(p) \text{Exp}(\mathbf{A}(p)\Delta) \quad (3)$$

$$\mathbf{P}_y(\omega) = \left| \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{-i\omega\tau} \rho_y(\tau) d\tau \right|^2 = \left| -\frac{1}{2\pi} (i\omega - \mathbf{A}(p))^{-1} \mathbf{v} \mathbf{I} (i\omega + \mathbf{A}^T(p))^{-1} \right|^2 \quad (4)$$

The following figures show the behavior of the autocorrelation and power spectrum as the system approaches instability.

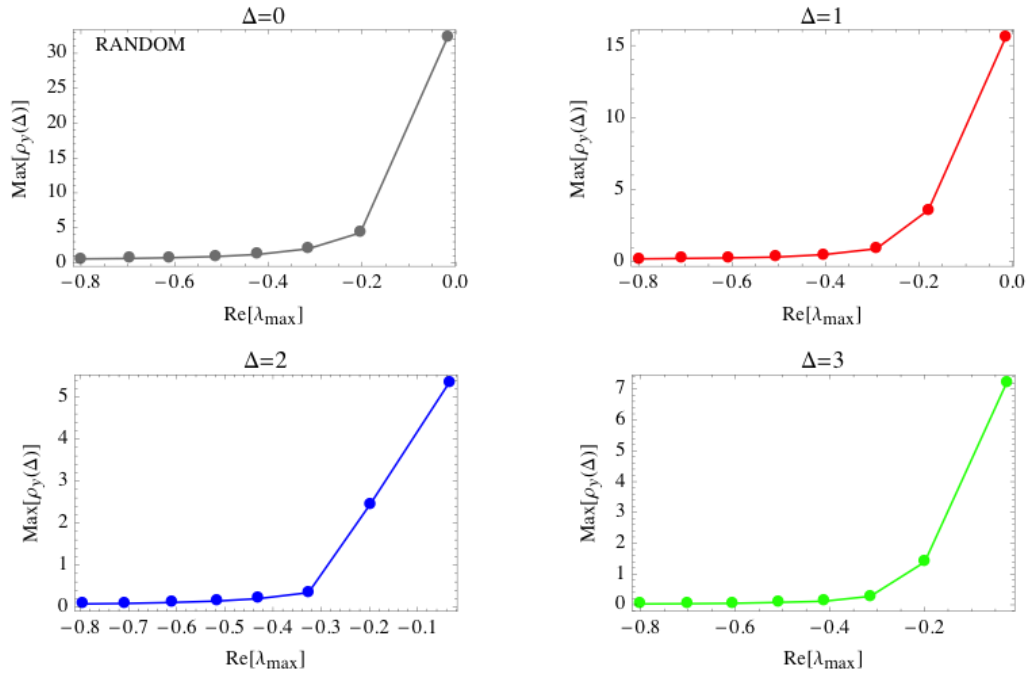


Figure S12. Increase in $\text{Max}[\rho_y]$ as $\text{Max}[\text{Re}(\lambda)]$ tends to zero for “strongly” disordered networks with a random architecture with $N=20$ and $C=0.3$. Increasing values of $\text{Max}[\text{Re}(\lambda)]$ are obtained by increasing the interaction strength, p . The plotted values are ensemble averages of 100 realizations.

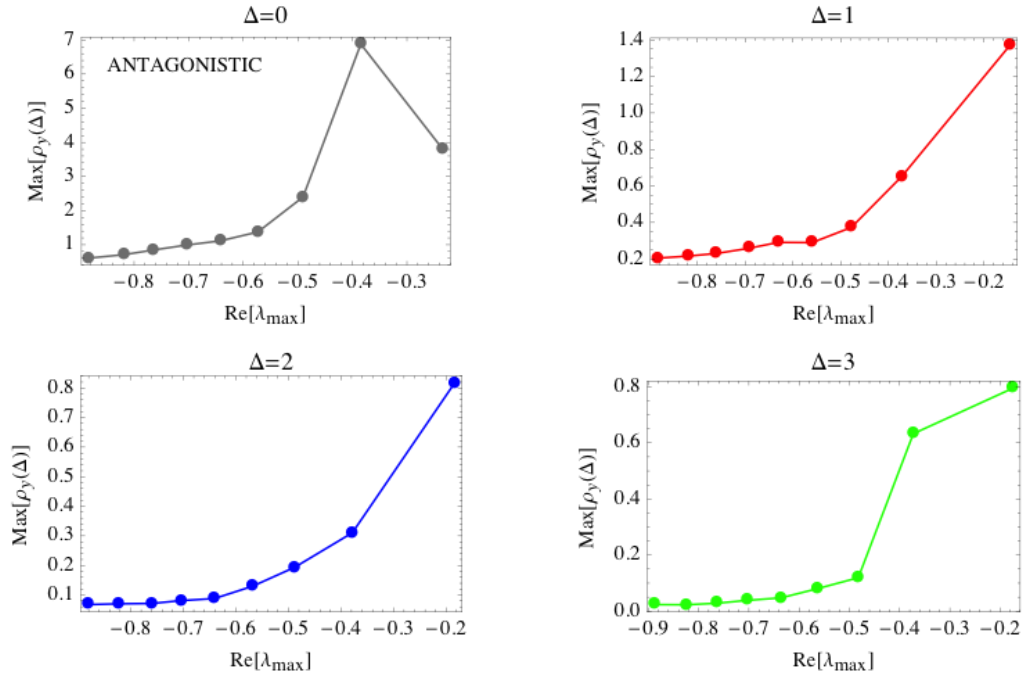
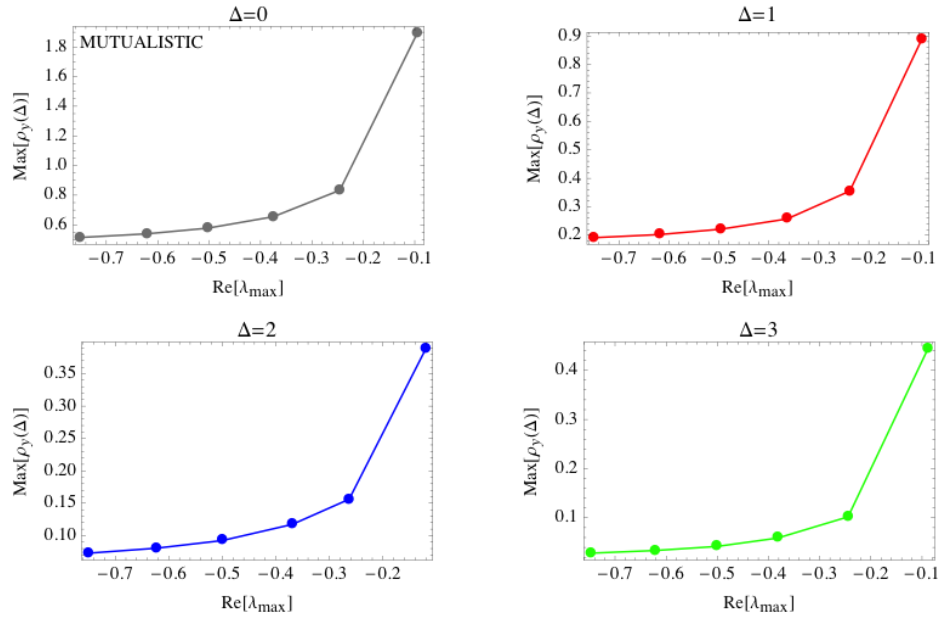


Figure S13. Increase in $\text{Max}[\rho_y]$ as $\text{Max}[\text{Re}(\lambda)]$ tends to zero for “strongly” disordered networks with a predator-prey architecture and $N=20$, $C=0.3$. Increasing values of $\text{Max}[\text{Re}(\lambda)]$ are obtained by increasing the interaction strength, p . The plotted values are ensemble averages of 100 realizations.



191

192 **Figure S14.** Increase in $\text{Max}[\rho_y]$ as $\text{Max}[\text{Re}(\lambda)]$ tends to zero for “strongly” disordered
 193 networks with a mutualistic architecture and $N=20$, $C=0.3$. Increasing values of $\text{Max}[\text{Re}(\lambda)]$
 194 are obtained by increasing the interaction strength, p . The plotted values are ensemble
 195 averages of 100 realizations.

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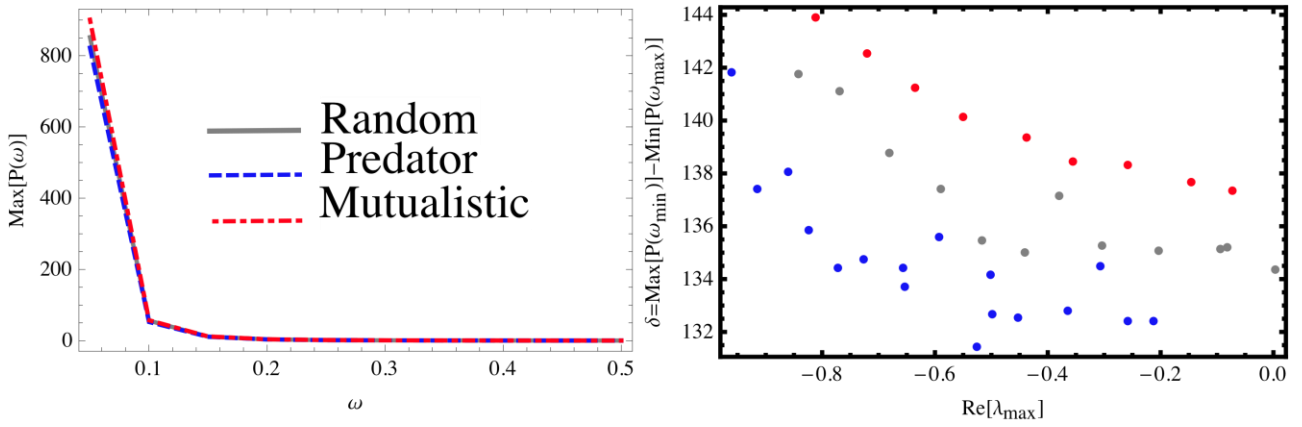


Figure S15. Left panel: max element of the power spectrum matrix as a function of frequency for three different architectures. The impact of the structure on the spectrum is negligible. Right panel: Power spectrum evaluated in the minimum and maximum frequency as p tends to p_c (and thus $\text{Max}[\text{Re}(\lambda)]$ tends to zero) for strongly disordered systems ($N=20$ and $C=0.2$) with random, predator-prey, and mutualistic interactions. Increasing values of $\text{Max}[\text{Re}(\lambda)]$ lead to a decrease in $\delta = \text{Max}[P(\omega_{\min})] - \text{Max}[P(\omega_{\max})]$, that therefore might be considered a precursor for a critical transition. However, the intensity of this early warning sign is quite weak and thus difficult to detect. The plotted values are ensemble averages of 100 realizations.

S5. Early Warning Detection

To evaluate whether the onset to instability can be anticipated in time by an increase in $Max[\mathbf{S}_y]$ (or in other suitably chosen elements of \mathbf{S}_y), we test the correlation (11) between $Max[\mathbf{S}_y]$ and the control parameter (p or C) that is gradually varied to increase $Max[Re(\lambda)]$ up to a given threshold (here chosen equal to -0.2). If the correlation, ρ_k , (evaluated with the Kendall- τ test) is significant and greater than 0.5, the increase in $Max[\mathbf{S}_y]$ is interpreted as an early warning sign. We repeat this analysis for 1000 realizations of the random interaction strength network and determine the distribution of correlations along with the number of realizations with positive warning sign.

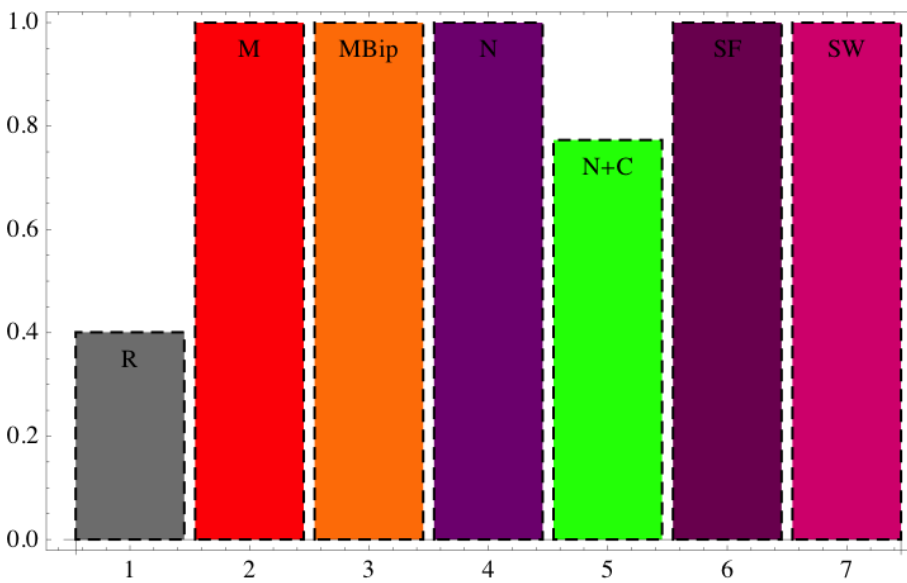


Figure S16. Probability of detecting true positives (i.e. of not missing a warning sign) in the case of mean field networks, using the same detection criteria as in Figure 4.

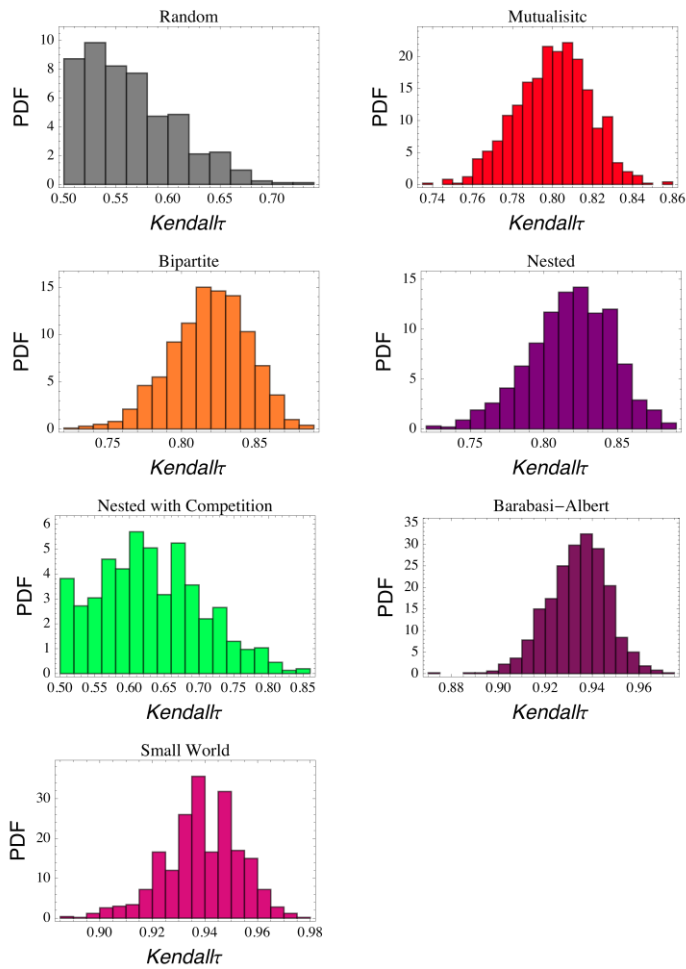


Figure S17. Frequency distribution of the ρ_K statistics used to detect early warning signs of instability in the case of mean field networks.

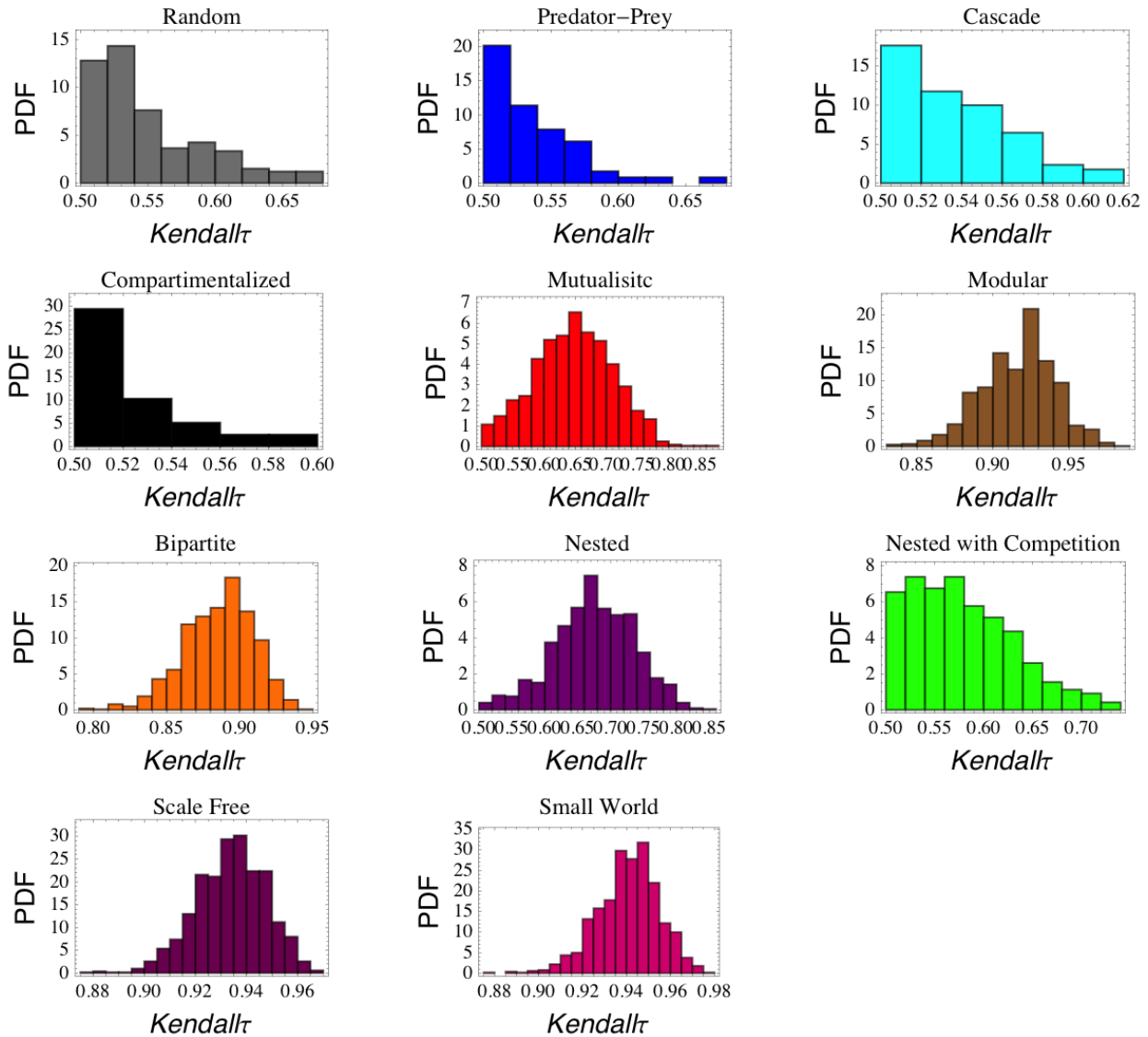


Figure S18. Frequency distribution of the ρ_K statistics used to detect early warning signs of instability in the case of full disordered networks.

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